INTERPOLATION IN ORLICZ AND SOBOLEV-ORLICZ SPACES

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§1. INTRODUCTION

The last two decades witnessed a rapid development of an umbrella-type theory of function spaces, in particular, of those frequently used in the theory of p.d.e.'s. The interpolation theory has become one of the powerful tools. A comprehensive monograph surveying results until the middle seventies is [T]. The classical interpolation theorems due to Riesz, Thorin, and Marcinkiewicz had become the basis for the complex and real interpolation method, resp. We shall restrict ourselves to the real method in virtue of the fact that only this method gives finer results in the framework of L_p-spaces.

Let X_1 and X_2 be B-spaces embedded into some Hausdorff topological space A. As usual let $\Delta(\bar{X}) = X_1 \cap X_2$ with the norm $\|x\|_\Delta = \max(\|x\|_{X_1}, \|x\|_{X_2})$ and $\Sigma(\bar{X}) = X_1 + X_2$ endowed with the norm $\|x\|_\Sigma = \inf\{\|x_1\|_{X_1} + \|x_2\|_{X_2}$; $x = x_1 + x_2$, $x_i \in X_i$, $i = 1,2\}$. Any B-space X such that $\Delta(\bar{X}) \subset X \subset \Sigma(\bar{X})$ (with a continuous embedding) is said to be an *interpolation space* between X_1 and X_2 if for each linear operator $T : \Sigma(\bar{X}) \to \Sigma(\bar{X})$ such that $T = X_i \to X_i$ and $T|_{X_i}$ is bounded, $i = 1,2$, T maps X into X and is bounded here. *An interpolation method* is a mapping $\mathcal{T}$ from the family $C(A)$ of all couples $\bar{X} = (X_1,X_2)$ with properties as above into the set of all B-spaces embedded into A which satisfies the following condition: Whenever $\bar{X}$, $\bar{Y} \in C(A)$ and T is a linear operator