# SOME PROBLEMS OF SPECTRAL THEORY

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An operator T, in a finite-dimensional space, is of scalar type - its matrix is diagonal in suitable coordinates - if and only if there exist numbers $\lambda_j$ and pairwise disjoint projections $P_j$ such that

$$T = \sum_j \lambda_j P_j . \tag{1}$$

Also compact operators of scalar type in any Banach can be so expressed, provided, the sum (1) is allowed to be countably infinite.

It is perhaps less often noted that any operator of scalar type can be expressed in the form (1), with the index j running over all positive integers. But, in general, the projections $P_j$ cannot be chosen pair-wise disjoint, that is, the product of any distinct pair of them might not be equal to the zero-operator.

In fact, an operator T in a space E is of scalar type if and only if, there exists an abstract space $\Omega$, a $\sigma$-algebra $\mathcal{S}$ of its subsets, a $\sigma$-additive and multiplicative measure $P : \mathcal{S} \to L(E)$ such that $P(\Omega) = I$, the identity operator, and a P-integrable function f such that

$$T = \int f dP . \tag{2}$$

It is then easy to see that there exist sets $X_j \in \mathcal{S}$ and numbers $\lambda_j$ such that (1) holds with $P_j = P(X_j)$, $j = 1,2,\ldots$.

Now, it might very well happen that an operator T can be expressed in the form (1) but it is not possible to guarantee that the projections $P_j$ are values of a spectral measure P such that (2) holds. To be sure, the expression (1) is of little value if nothing is known about the operators $P_j$ save that they are projections; some additional structure