

THE INHERITANCE OF CONCAVITY BY
SOLUTIONS OF BOUNDARY VALUE PROBLEMS

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1. INTRODUCTION

A significant proportion of the literature dealing with boundary value problems is concerned with regularity inheritance results: under appropriate regularity assumptions for the data of the problem (the boundary, boundary function, differential operator and "force" function) one attempts to demonstrate the regularity of the solution. Very little, however, has been published concerning the inheritance of various concavity-like geometrical properties. In this paper, a very brief and incomplete history of the topic is outlined, after which a recent result of the author is sketched.

2. DEFINITIONS

The concept of α -concavity permits a unified presentation of concavity inheritance results. It is adapted from a definition of Brascamp and Lieb ([1]).

Let K be a convex subset of \mathbb{R}^n , and u be a positive real-valued function on K . Then for $\alpha > 0$, u is said to be α -concave when u^α is concave. Extending this definition in a natural way, one says that u is α -concave for $\alpha < 0$ when u^α is convex, and 0-concave (or *log concave*) when $\log(u)$ is concave. As a further extension, one may say that u is $(-\infty)$ -concave (or *pseudo-concave*) when