WEAKLY INTEGRABLE SEMIGROUPS

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The connection between Markov processes and semigroups of continuous linear operators is now well established. However, early in the development of the theory William Feller observed that the use of strongly continuous semigroups is not always appropriate [3].

Subsequently, the interests of probabilists turned towards other directions and to new methods. As a consequence, there still seems to be no coherent account of the class of semigroups most appropriate for the study of Markov processes and diffusion processes. The work of E.B. Dynkin [2] is suggestive in this context, but it is not sufficiently developed. The approach suggested here is based on a number of principles in common with many problems in analysis.

Weak (Pettis-type) integration, as opposed to the strong (Bochner) integrals used in the standard theory is the natural tool for this situation. Secondly, it is often necessary to weaken the topology of the underlying vector space, even to utilize the weak integral - a feature in common with the spectral theory of operators [5]. Another aspect of the present approach is that the "infinitesimal generator" of the semigroup is defined directly in terms of the resolvent, instead of by differentiation as is most commonly done; this has the advantage of substituting integral operators and equations for differential ones - a time-honoured and successful method in analysis [1;p.207].

Characteristic examples of the type of semigroups treated here arise from diffusion processes generated by an elliptic differential operator L