$C^{1,1}$-REGULARITY OF SOLUTIONS TO VARIATIONAL INEQUALITIES

Gerhard Huisken

INTRODUCTION

Let $\Omega \subset \mathbb{R}^n$, $n \geq 2$ be an open bounded set with smooth boundary and let u be a solution of a variational inequality of the form

(1) $$u \in K_1 \; ; \; \langle Au+H(x,u,Du), v-u\rangle \geq 0 \qquad \forall v \in K_1$$

where A is a quasilinear elliptic operator in divergence form

(2) $$Au = -D_i(a^i(x,u,Du))$$ [1)]

and the convex set K_1 is given by

(3) $$K_1 = \{v \in H^{1,\infty}(\Omega) \,|\, v \geq \psi, v|_{\partial\Omega} = \phi\} .$$

We also consider the case of Neumann boundary conditions, i.e. solutions of

(4) $$u \in K_2 \; ; \; \langle\langle Au + H(x,u,Du) \; , \; v-u\rangle\rangle \geq 0 \qquad \forall v \in K_2$$

(5) $$K_2 = \{v \in H^{1,\infty}(\Omega) \,|\, v \geq \psi\}$$

where

(6) $$\langle\langle Au,\eta\rangle\rangle = \int_\Omega a^i(x,u,Du)\cdot D_i\eta dx + \int_{\partial\Omega} \beta\cdot\eta dH_{n-1}$$

for some function β on $\partial\Omega$.

---

1) Here and in the following we sum over repeated indices.