COMPUTING FOURIER AND LAPLACE TRANSFORMS BY MEANS OF POWER SERIES EVALUATION

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1. NOTATIONS AND ASSUMPTIONS

Let f be a real-valued function, defined for nonnegative arguments. We shall discuss some aspects of the numerical evaluation of the Laplace transform

$$(Lf)(\lambda) = \int_0^\infty e^{-\lambda t} f(t)\,dt \,, \tag{1.1}$$

and the Fourier transform

$$(Ff)(\omega) = \int_0^\infty e^{i\omega t} f(t)\,dt \,. \tag{1.2}$$

It will turn out to be advantageous to treat (1.1) and (1.2) separately, even if (1.2) is obtained by setting $\lambda = -i\omega$ in (1.1). We shall confine our discussion to the cases λ and ω real. We observe that the two-sided Fourier transform can be cast on the form of (1.2) since

$$\int_{-\infty}^\infty e^{i\omega t} f(t)\,dt = \int_0^\infty e^{i\omega t} f(t)\,dt + \int_0^\infty e^{-i\omega t} f(-t)\,dt \,.$$

Therefore, the inverse Laplace transform may be calculated by means of evaluating integrals of the type of (1.1) and (1.2). (See e.g. [1] and [4].) In our treatment we shall assume that $f(t)$ may be calculated for an arbitrary argument t with known, finite accuracy. In order to assess the accuracy of the calculated values of (1.1) and (1.2) we must know that f