

BOOLEAN ALGEBRAS OF PROJECTIONS
IN
LOCALLY CONVEX SPACES

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INTRODUCTION

The object of this paper is to report recent work with Werner Ricker on algebras of scalar-type spectral operators in locally convex spaces. The details will appear elsewhere [3]. Our motivation lies in the penetrating study by W.G. Bade [1], [2] (see also [4]) of operator algebras generated by Boolean algebras of projections in Banach spaces. At the heart of Bade's work lies the Bartle - Dunford - Schwartz theory of Banach space-valued measures. Consequently, a natural avenue of approach in the present setting is via the study of spectral measures in locally convex spaces, a study initiated by Schaefer [11] and further developed by Walsh [12] and, more recently, by Ricker [9]. In the study of operator algebras, there are, however, new difficulties which arise in the locally convex setting. In the resolution of these difficulties, a decisive role is played by the notion of closed vector measure, introduced by Kluvánek [5].

1. BOOLEAN ALGEBRAS OF PROJECTIONS

To fix notation, X will denote a locally convex space which is assumed to be quasi-complete. $L(X)$ will denote the space of continuous linear operators on X equipped with the topology of pointwise convergence on X . $L(X)$ is assumed to be sequentially complete. A subset \mathfrak{B} of $L(X)$ is called equicontinuous if and only if for each continuous semi-norm q on X , there exists a continuous semi-norm p on X such that