THE DIRICHLET PROBLEM FOR A LINEAR ELLIPTIC EQUATION IN A HALF SPACE WITH L^2-BOUNDARY DATA

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Let $R_n^+ = \{x \,;\, x \in R_n \,,\, x_n > 0\}$. We denote point $x \in R_n^+$ by $x = (x', x_n)$, where $x' = (x_1, x_2, \ldots, x_{n-1}) \in R_{n-1}$.

We consider the Dirichlet problem for the elliptic equation of the form

$$Lu = -\sum_{i,j=1}^{n} D_i(a_{ij}(x)\, D_j u) + \sum_{i=1}^{n} b_i(x)\, D_i u + c(x)\, u = f(x) \tag{1}$$

in R_n^+ . We make the following assumptions about the operator L :

(A) L is uniformly elliptic in R_n^+ , i.e., there exists a positive constant δ such that

$$\delta|\xi|^2 \le \sum_{i,j=1}^{n} a_{ij}(x)\, \xi_i\, \xi_j$$

for all $x \in R_n^+$ and $\xi \in R_n$, moreover $a_{ij} \in L^\infty(R_n^+)$ $(i,j = 1, \ldots, n)$.

(B) (i) There exist positive constants K and $0 < \alpha < 1$ such that

$$|a_{nn}(x', x_n) - a_{nn}(x', \bar{x}_n)| \le K|x_n - \bar{x}_n|^{\alpha}$$

for all $x' \in R_{n-1}$ and all x_n , $\bar{x}_n \in [0,\infty)$.

(ii) $a_{in} \in C^1(R_n^+)$ and $|D_k\, a_{in}(x)| \le K_1\, x_n^{-\beta}$ for all $x \in R_{n-1} \times (0,b]$, where K_1 , b and β are positive constants,