THE GROUP OF INVERTIBLE ELEMENTS OF CERTAIN BANACH ALGEBRAS OF OPERATORS ON HILBERT SPACE

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0. INTRODUCTION

If E denotes a real or complex Hilbert space, J a complex structure on E and $\mathfrak{S}$ a separable symmetrically normed ideal in the algebra $\mathcal{B}(E)$ of bounded operators on E then one may define a subalgebra $\mathcal{B}_{\mathfrak{S}}(E)$ of $\mathcal{B}(E)$ by

$$\mathcal{B}_{\mathfrak{S}}(E) = \{A \in \mathcal{B}(E) | AJ - JA \in \mathfrak{S}\} .$$

Then $\mathcal{B}_{\mathfrak{S}}(E)$ may be normed to become a Banach algebra. The homotopy type of the group $G_{\mathfrak{S}}(E)$ of invertible elements of $\mathcal{B}_{\mathfrak{S}}(E)$ may be determined (it is the same as that of a classifying space for a certain functor of K-theory). When $\mathfrak{S} = \mathfrak{S}_2$ (the Hilbert-Schmidt ideal) the orthogonal or unitary retracts of $G_{\mathfrak{S}}(E)$ have a physical interpretation in terms of automorphisms of the infinite dimensional Clifford algebra. Moreover the first K-group, for E real, $K_1\left(\mathcal{B}_{\mathfrak{S}_2}(E)\right) \cong \mathbb{Z}_2$, relates to the existence of two distinct phases in the Ising model below the critical temperature and for E complex, $K_1\left(\mathcal{B}_{\mathfrak{S}_2}(E)\right) \cong \mathbb{Z}$, may be interpreted in terms of the electric charge in the second quantised Dirac theory of the electron.