BOX MAXIMAL FUNCTIONS*

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The theory of Hardy spaces is closely intertwined with the study of partial differential equations. Properties of analytic and harmonic functions, and temperatures, are key ingredients in proving basic results concerning Hardy spaces. These results in turn establish new principles which can be used in the study of p.d.e.'s. It is also often the case that, in spite of their name, harmonic analysts prefer proofs concerning the Hardy spaces independent of properties of analytic or harmonic functions.

We will illustrate some of these remarks by presenting an elementary proof of an estimate involving box maximal functions. Our strategy will be to view this result as an imbedding inequality of Hardy and Littlewood type and adapt to this setting the techniques introduced by Calderón and Torchinsky [2, Lemma 2.6] and Jawerth and Torchinsky [7]. Our result will extend an estimate which appears in recent work of Chanillo and Wheeden [3], [4] as a step in obtaining weighted Poincaré and Sobolev inequalities.

First some background. While discussing a basic result in the theory of Hardy spaces of several real variables, namely the passage to arbitrary approximate identities in the definition of the $H^p(R^n)$ spaces, C. Fefferman and Stein [6] introduced the following box maximal function. Let $T(x,y) = \{(y,s) \in R^{n+1}_+ : |x-y| \le h , 0 < s \le h\}$ denote the box over x of height h ; and let f be defined on R^n and u its Poisson integral in the upper half-space. Then set

* Research conducted at the Australian National University under the sponsorship of a grant from the Centre for Mathematical Analysis.