LATTICES FOR MULTIPLE INTEGRATION

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1. INTRODUCTION

The subject of this paper is the numerical integration of smooth, periodic functions over the unit cube in s dimensions. In one dimension the problem reduces to

$$\int_0^1 f(x)dx \ , \tag{1}$$

where f is a smooth, periodic function with period 1. In this situation the recommended method (see for example [3, p. 106]) is usually the trapezoidal rule, or equivalently the rectangle rule

$$\frac{1}{n}\sum_{j=0}^{n-1} f\left(\frac{j}{n}\right) \ . \tag{2}$$

The s-dimensional analogue of (1) is

$$\begin{aligned} I(f) &= \int_0^1 \cdots \int_0^1 f(x_1, \ldots, x_s)dx_1 \cdots dx_s \\ &= \int_{U^s} f(\underset{\sim}{x})d\underset{\sim}{x} \ , \end{aligned} \tag{3}$$

where U^s is the s-dimensional unit cube,

$$U^s = \{\underset{\sim}{x} \in \mathbb{R}^s : \ 0 \le x_i < 1 \ , \ 1 \le i \le s\} \ .$$

We shall always assume that f is periodic with period 1 with respect