COLLOCATION METHODS FOR SECOND KIND FREDHOLM INTEGRAL EQUATIONS

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1. INTRODUCTION

In this paper we consider the application of the collocation method and its iterated variant to the numerical solution of the Fredholm integral equation

$$(1.1) \qquad y(t) = f(t) + \lambda \int_0^1 k(t,s)y(s)ds \ , \quad t \in [0,1] \ ,$$

where f and k are known, λ is a given scalar and y is the solution to be determined. The equation can be written in operator notation as

$$y = f + \lambda Ky \ .$$

Taking C to be the Banach space of continuous functions on $[0,1]$ equipped with the uniform norm, we shall make the following assumptions on (1.1):

A1: $f \in C$;

A2: K is a compact operator from C to C ;

A3: the homogeneous equation $y = Ky$ has only the trivial solution.

It then follows from standard Fredholm theory that there exists a unique solution $y \in C$.

In the collocation method, y is approximated by a function belonging to some finite-dimensional subspace taken here to be a space of discontinuous piecewise polynomials. It is well-known that under suitable