NUMERICAL METHODS FOR INVERSE EIGENVALUE PROBLEMS IN ALGEBRAIC CONTROL THEORY

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In this talk we outline three numerical methods for solving the following problem (details are to be reported elsewhere, see also [2]):

Given n linear subspaces $\mathcal{S}_j \subset E_n$ in the n-dimensional real vector space choose one vector $\underline{x}_j \in \mathcal{S}_j$, $j = 1,2,\ldots,n$ in each so that these n vectors $\underline{x}_1,\underline{x}_2,\ldots,\underline{x}_n$ are as orthogonal as possible.

Problems of this kind arise, for example, in algebraic control theory when, given an $n\times n$ matrix A , an $n\times m$ matrix B of rank m and numbers $\lambda_1,\lambda_2,\ldots,\lambda_n$ we seek an $m\times n$ matrix F such that the eigenvalues of the matrix $A+BF$ are the given numbers $\lambda_1,\ldots,\lambda_n$. For $m>1$ there may be many solutions F and it is then desirable to construct that F for which the eigenvalues λ_j of $A+BF$ are least sensitive to perturbations. This sensitivity is proportional to the condition numbers c_j (see [4]) of eigenvalues λ_j given by $c_j = \|\underline{x}_j\| \, \|\underline{y}_j\|$ where the right eigenvectors $\underline{x}_j$ satisfy

$$(1) \qquad (A - \lambda_j I + BF)\underline{x}_j = \underline{0}$$

and $\underline{y}_j$ are the left eigenvectors given by

$$\underline{y}_j = X^{-T}\underline{e}_j \quad , \quad j = 1,2,\ldots,n \; ,$$

where $X = (\underline{x}_1,\underline{x}_2,\ldots,\underline{x}_n)$. As $c_j \geq 1$ with equality (for all $j = 1,2,\ldots,n$) occuring iff the columns $\underline{x}_j$ of X are orthogonal there are many measures of the vector $\underline{c} = (c_1,\ldots,c_n)^T$ which can be minimized to express mathematically