THE SOLUTION OF SINGULAR-VALUE AND EIGENVALUE PROBLEMS ON SYSTOLIC ARRAYS

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0. SUMMARY

Parallel algorithms are presented for computing a singular-value decomposition of an $m \times n$ matrix $(m \ge n)$ and an eigenvalue decomposition of an $n \times n$ symmetric matrix. A linear array of O(n) processors is proposed for the singular-value problem and the associated algorithm requires time O(mnS), where S is the number of Jacobi sweeps (typically $S \le 10$). A square array of $O(n^2)$ processors with nearest-neighbor communication is proposed for the eigenvalue problem; the associated algorithm requires time O(nS).

1. INTRODUCTION

A singular-value decomposition (SVD) of a real $m \times n$ $(m \ge n)$ matrix A is its factorization into the product of three matrices:

$$(1.1) A = U \Sigma V^{T},$$

where U is an $m \times n$ matrix with orthonormal columns, Σ is an $n \times n$ nonnegative diagonal matrix and the $n \times n$ matrix V is orthogonal. This decomposition has many important scientific and engineering applications (cf. [1,11,27,28]).