SOLUTION OF DENSE SYSTEMS OF LINEAR EQUATIONS USING A FIXED SIZE SYSTOLIC ARRAY

A. Bojańczyk

1. INTRODUCTION

We consider the problem of parallel triangularization of a dense matrix of dimension $n \times m$, $n \leq m$. Matrix triangularization is a crucial step in solving dense systems of linear equations or dense linear least squares problems. Thus being able to accomplish such a transformation is very important in practice. However algorithms for matrix triangularization are computationally expensive. Hence there is a good reason to use a special purpose (VLSI) device for speeding-up such computations. Usually these devices implement systolic algorithms (Kung [4]) derived from classical direct methods. Two of the methods are particularly well-suited for our purpose. These are Gaussian elimination and QR decomposition by Givens rotations (see Kung and Leiserson [5], Bojańczyk, Brent and Kung [2], Gentleman and Kung [3]).

This paper presents a unified treatment of a systolic implementation of both Gaussian elimination with neighbour pivoting and QR decomposition. We also derive a scheme of splitting the matrix of coefficients in case a matrix is so large that it requires more processors than a given systolic array provides. Finally, we show how to handle banded matrices with $O(\text{bandwidth})^2$ processors.

2. PARALLEL TRIANGULARIZATION

Given an $n \times m$ matrix A of rank n , we want to determine an $n \times m$