# CORNER SINGULARITIES AND BOUNDARY INTEGRAL EQUATIONS

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## INTRODUCTION

It is well known that if $U$ is the solution to an elliptic boundary value problem (BVP) posed on a region $\Omega$, then, in general, $U$ will not be smooth at any corner point of $\Omega$. Because of this, special care is needed when designing numerical methods for solving the BVP, especially if high accuracy is needed near the corners, see e.g. [8]. As might be expected, similar difficulties arise when the BVP is reformulated as a boundary integral equation (BIE), that is, the solution to the BIE fails, in general, to be smooth at the corner points of the boundary.

In this paper we discuss the double layer potential equation for a polygon, a BIE which occurs as a reformulation of the Dirichlet problem for Laplace's equation. With this simple example, we illustrate some of the difficulties created by the presence of corners, and how one might deal with them. A more detailed technical discussion of this and similar problems can be found in papers such as [2], [3], [4] and [5].

## 1. THE INTEGRAL EQUATION

Let $\Omega$ be a polygonal domain. That is, $\Omega$ is a bounded, connected, open subset of $\mathbb{R}^2$ whose boundary $\Gamma = \partial\Omega$ is made up of a finite number of straight line segments. Let $x_1, x_2, \ldots, x_N$ be the vertices of $\Omega$, numbered so that $x_{j+1}$ follows $x_j$ as one proceeds anticlockwise around $\Gamma$. We adopt the convention that subscripts are evaluated modulo $N$