THE BOUNDARY INTEGRAL METHOD FOR PDE's.

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The last ten years have seen the development of the boundary integral method as an important tool in practical engineering computations The early work in aeronautical fluid flow (see Hess [1975]) elastostatics (see Brebbia [1978], Cruse and Rizzo [1968]) and potential theory (see Jaswon and Symm [1977]) has lead to the sophisticated techniques and varied applications reported more recently in Brebbia, Futagami, and Tanaka [1983], Brebbia, Telles and Wrobel [1983], Liggett and Liu [1983], and Butter et al [1983] . Here we give an introduction to these ideas.

1. POTENTIAL PROBLEMS

The easiest application of the boundary integral method is to the two dimensional problems of classical potential theory (Kellog [1929], Mikhlin [1970]). We are given an open bounded region $\Omega \subset \mathbb{R}^2$ whose boundary Γ is smooth except for a finite number of corners. We need the solution U of the partial differential equation

(1) $$\Delta U(x) = \frac{\partial^2 U}{\partial x_1^2} + \frac{\partial^2 U}{\partial x_2^2} = 0 \qquad , x = (x_1,x_2) \in \Omega$$

with the boundary conditions

(2.1) $$U(x) = g(x) , \qquad x \in \Gamma_0$$

(2.2) $$U_\nu(x) = h(x) , \qquad x \in \Gamma_1$$

Here for any $x \in \Gamma$ (except a corner point) $\nu(x)$ is the outward normal and $U_\nu(x) = \nabla U(x).\nu(x)$ is the normal derivative. Γ_0 and Γ_1 are the disjoint components of the boundary on which Dirichlet data g and Neumann data h are given. (For the Neumann problem with $\Gamma_1 = \Gamma$ we need $\int_\Gamma h = 0$ and impose the extra condition $\int_\Gamma U = 0$, on the solution). These are the simplest models used in heat transfer and fluid flow calculations.