NONCONFORMING ELEMENTS, PATCH TESTS AND INTER-ELEMENT CONSTRAINTS

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1. INTRODUCTION

For a variational problem requiring that the admissible functions have partial derivatives of order m that are square integrable, it is sufficient that the piecewise polynomials in the finite element basis have continuous derivatives up to order $m - 1$. That is, $v_h \in C^{m-1}$ where h denotes the mesh parameter for a finite element discretization of the domain. Hence for second order problems ($2m = 2$) the basis is C^0 (globally continuous) while for fourth-order problems ($2m = 4$) it is C^1 (derivatives to first order are continuous). Such finite element bases are said to be conforming. If $v_h \notin C^{m-1}$ the basis is said to be nonconforming.

Because of their simplicity and relatively low degree, nonconforming finite elements were first applied to finite element calculations of plate bending problems in the mid 1960's. These exploratory studies produced conflicting results - in some instances they led to accurate converging solutions, while in other calculations the method failed to converge. In particular, for certain test problems with nonconforming elements it was observed that the method converged for some mesh orientations but not for others (Bazeley et al. [1965]).

This sensitivity to mesh orientation led to the proposition of a numerical "patch test" by Irons (see Irons and Razzaque [1972]). The essential idea of their patch test was that in the limit as the mesh size approaches