ANALYSIS OF EXPLICIT FINITE DIFFERENCE METHODS USED IN COMPUTATIONAL FLUID MECHANICS

John Noye

1. INTRODUCTION

It is now commonplace to simulate fluid motion by numerically solving the governing partial differential equations on high speed digital computers.

Finite difference techniques, because of their relative simplicity and their long history of successful application, are the most commonly used. They have, for example, been used in depth-averaged and three-dimensional time dependent tidal modelling by many oceanographers and coastal engineers : see, for example, Noye and Tronson (1978), Noye et.al. (1982) and Noye (1984a).

However, like finite element techniques and boundary integral methods, finite difference methods of solving the Eulerian equations of hydrodynamics seldom model the advective terms accurately. Errors in the phase and amplitude of waves are usual, particularly the former.

The accuracy of various explicit finite difference methods applied to solving the advection equation, namely

$$(1.1) \qquad \frac{\partial\bar{\tau}}{\partial t} + u\frac{\partial\bar{\tau}}{\partial x} = 0,\ 0 \le x \le 1,\ t > 0,\ u \text{ a positive constant,}$$

is investigated in this work. The boundary condition to be used in practice is that $\bar{\tau}(0,t)$ is defined for $t > 0$, with no values prescribed at $x = 1$.

The von Neumann amplication factor is not only used to find the stability criteria of the methods investigated, but also to determine the wave deformation properties of the technique. These properties are then linked to the "modified" equation; that is, the partial differential equation which is equivalent to the finite difference equation, after the former has been modified so it contains only the one temporal derivative, $\partial\bar{\tau}/\partial t$, all other derivatives being spatial.

It will be seen that successively more accurate methods can be developed by systematic elimination of the higher order terms in the truncation error, which is the difference between the modified equation and the given equation (1.1).