PROBLEMS WITH DIFFERENT TIME SCALES

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1. INTRODUCTION

Perhaps the simplest problem with different time scales is given by the initial value problem for the ordinary differential equation

$$(1.1)\qquad \varepsilon dy/dt = ay + e^{it}, \quad t \geq 0, \quad y(0) = y_0 .$$

Here ε, a are constants with $0 < \varepsilon << 1$, $|a| = O(1)$ and Real $a \leq 0$. The solution of (1.1) is given by

$$(1.2)\qquad y(t) = y_S(t) + y_R(t),$$

where

$$y_S(t) = e^{it}(-a+i\varepsilon)^{-1}, \quad y_R(t) = e^{(a/\varepsilon)t}(y_0 - y_S(0)) .$$

Thus it consists of the slowly varying part $y_S(t)$ and the rapidly changing part $y_R(t)$. There are two fundamentally different situations

1) <u>$a = -1$</u>. In this case $y_R(t)$ decays rapidly and outside a boundary layer the solution of (1.1) varies slowly. Many people have developed numerical methods to solve problems of this kind (see for example [15]) and we shall not consider this case.

2) <u>$a = i$ is purely imaginary</u>. Now $y_R(t)$ does not decay and $y(t)$ is highly oscillatory everywhere. In many applications one is not interested in the fast time scale. Therefore it is of interest to