

THE FACTORIZATION METHOD FOR TWO POINT BOUNDARY VALUE PROBLEMS
FOR ODE'S AND ITS RELATION TO THE FINITE DIFFERENCE METHOD

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1. INTRODUCTION

Finite difference and finite element methods for solving two point boundary value problems for systems of ordinary differential equations consist of

- (a) a discretization procedure which transforms the original problem into a family of finite dimensional systems of algebraic equations parametrized by the mesh size h , and
- (b) a solution procedure for the systems of algebraic equations.

For linear boundary value problems the algebraic equations are linear and step (b) reduces to the selection of a matrix reduction scheme. In this paper we consider only direct (elimination) methods of matrix reduction.

By these two steps, taken together, the original problem is transformed into a sequential numerical process (§5) which depends on the mesh parameter h . A complete analysis of the numerical procedure must consider this underlying numerical process, not merely the discretization step (a). In this paper we carry out such a complete analysis for a model singular perturbation problem of turning point type (§2) studied by H.O. Kreiss et al. in [1]. We show (§4) that the numerical process converges, as $h \rightarrow 0$, to the solution of initial value problems for certain differential equations. These limiting equations are the closure [2] of the process. Thus it is