SINC METHODS OF APPROXIMATE SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS

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1. INTRODUCTION AND SUMMARY

In the author's experience, nearly all solutions of PDE (partial differential equations) encountered in applications are piecewise analytic in each variable. Except in the case of inverse problems, we can predict a priori the regions of analyticity of the solutions of PDE. The solutions of linear PDE are analytic whenever the coefficients of the PDE are analytic, although singularities may also occur on the boundary of the region.

In this paper we derive two families of methods for solving second order PDE. Each of these families is based on the Whittaker cardinal function, or sinc function expansion of a function f defined on the real line $\mathbb{R}$. This expansion takes the form

$$C(f,h) = \sum_{k\in\mathbb{Z}} f(kh)\,S(k,h) \tag{1.1}$$

where $h>0$, $\mathbb{Z} = \{0,\pm1,\pm2,\ldots\}$, and where the sinc function $S(k,h)$ is defined by

$$S(k,h)\circ(x) = \frac{\sin\{\frac{\pi}{h}(x-kh)\}}{\frac{\pi}{h}(x-kh)}\ , \quad k\in\mathbb{Z}\ . \tag{1.2}$$

While formulas for approximating derivatives of function defined on $\mathbb{R}$ are immediately obtainable from (1.1), we