

# THE ROLE OF ANALYSIS IN THE SOLUTION OF PARTIAL DIFFERENTIAL EQUATIONS

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## 1. INTRODUCTION

By definition, the end result of a numerical calculation is one or more numbers. From a practical point of view, such numbers are not particularly useful unless they can be related back to the 'real world'. Therefore, when discussing the numerical solution of partial differential equations (or for that matter any mathematical equations), it is not unreasonable to focus attention on problems which attempt to model some physical, biological or social phenomenon. We shall in any case do so here.

Given that our equation (or equations) do indeed fall into this category, we might ask what role mathematical analysis plays in obtaining a solution to the problem. It is sometimes argued that questions such as existence and uniqueness are superfluous. After all, the phenomenon in question actually occurs in the real world. Clearly then, the model equations should also have a solution. As for the calculation of a numerical solution, surely the basic principles and intuition available about the subject matter in hand are a sufficient guide to achieve a satisfactory answer.

Unfortunately, "life wasn't meant to be easy" and the above approach to computational problem solving can (and often does) fail (though it must be conceded that many problems have been and will continue to be solved satisfactorily in this manner). Often, some mathematical analysis of the problem and the numerical scheme for its approximate solution, is an essential ingredient in obtaining a meaningful result. Conversely, the