BEST APPROXIMATION OPERATORS IN FUNCTIONAL ANALYSIS

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The purpose of this primarily expository talk is to introduce two classes of (generally non-linear) maps which occur naturally in functional analysis. Both classes consist of retracts from a given Banach space E onto a closed subspace M.

The first class is that of best approximation operators, or closest point maps. There is already an enormous amount known about this subject. Our coverage is necessarily brief, so for further information we refer to [4] and the references therein.

The second class is a generalization of the projections onto complemented subspaces. Since the orthogonal projection onto a subspace of a Hilbert space is also the best approximation operator, these two classes do have something in common.

Given $a \in E$, we let $P(a) = P_M(a)$ be the set of points in M which best approximate a. That is,

$$P(a) = \{x \in M; \|x - a\| = d(a, M)\} = M \cap B(a, d(a, M)) .$$

If $P(a)$ contains exactly (at least/at most) one element, for every