ITERATIVE METHODS FOR SOME LARGE SCALE GENERALIZED EQUATIONS

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1. INTRODUCTION

Let T be a set valued mapping (multifunction) of $\mathbb{R}^n$ into $\mathbb{R}^n$ (that is $T(x) \subseteq \mathbb{R}^n$ for all $x \in \mathbb{R}^n$). Consider the problem of finding a zero of the map T, that is a point $x^* \in \mathbb{R}^n$ which satisfies the *generalized equation*

$$0 \in T(x^*) \quad (1.1)$$

Such problems frequently arise as necessary conditions for optimization problems, so the continuity properties of the solution sets are of considerable interest (see [3] for example). However here the interest is in numerical methods for calculating x^*. In particular attention is restricted to those maps T where the problem of finding x^* is equivalent to the problem of minimizing some function F.

Consider the maps T which are the generalized gradient $\partial F(x)$ of a locally Lipschitz function $F:\mathbb{R}^n \to \mathbb{R}$. For any $x \in \mathbb{R}^n$ $\partial F(x)$ is a non-empty compact convex set in $\mathbb{R}^n$, and the mapping ∂F is upper semi-continuous (Clarke [1]). A locally Lipschitz function is differentiable almost everywhere, and $\partial F(x)$ is a singleton (the gradient of F) if and only if F is differentiable at x. Also if F is convex then the generalized gradient is the subdifferential of F. A necessary condition for a point x^* to be a local minimizer of F is that $0 \in \partial F(x^*)$, so solving (1.1) equivalent to finding a stationary point of F.