EXISTENCE VIA INTERIOR ESTIMATES FOR SECOND ORDER PARABOLIC EQUATIONS

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In memory of a former student of J.H. Michael, the late
Robin Wittwer (17th February 1954 - 26th May 1984)

1. PRELIMINARIES

Our problems will be solved on subsets of $\mathbb{R}^{n+1}$ with $n \geq 1$. We label points X in $\mathbb{R}^{n+1}$ by (x,t), $x \in \mathbb{R}^n$, $t \in \mathbb{R}$, the $(n+1)$-th component being often associated with time in physical problems. For $X = (x,t)$, we call $|X| = (\|x\|^2 + |t|)^{\frac{1}{2}}$, the parabolic length of X, $\|x\|^2 = \sum_{i=1}^{n} x_i^2$ if $x = (x_1,\ldots,x_n)$. For $X,Y \in \mathbb{R}^{n+1}$, $d(X,Y) = |X-Y|$ denotes the parabolic distance between X and Y. Let Ω be a domain in $\mathbb{R}^{n+1}$. A point X in the topological boundary $\partial\Omega$ of Ω belongs to the parabolic boundary $P\Omega$ of Ω if for some $Y \in \Omega$, there exists a continuous path connecting X and Y, along which the "time" coordinate is non-decreasing. If $X \in \Omega$, then $d_\Omega(X)$ denotes $\inf\{d(X,Y);\ Y = (y,\tau) \in P\Omega,\ \tau \leq t\}$ if X is the point (x,t).

2. LINEAR PARABOLIC PARTIAL DIFFERENTIAL EQUATIONS

Linear parabolic partial differential operators will be defined on functions u defined on domains Ω to have the following form:

$$Lu(X) \equiv \sum_{i,j=1}^{n} a_{ij}(X) \frac{\partial^2 u}{\partial x_i \partial x_j}(X) + \sum_{i=1}^{n} b_i(X) \frac{\partial u}{\partial x_i}(X) + c(X)u(X) - \frac{\partial u}{\partial t}(X)$$

for $X \in \Omega$, a_{ij}, b_i, c, being real valued, locally Hölder continuous