SOME RECENT RESULTS ON THE EQUATION OF PRESCRIBED GAUSS CURVATURE

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In this article we discuss some recently established results concerning convex solutions $u \in C^2(\Omega)$ of the equation of prescribed Gauss curvature

(1) $$\det D^2u = K(x)(1+|Du|^2)^{(n+2)/2} .$$

Here Ω is a domain in $\mathbb{R}^n$, Du and D^2u denote the gradient and the Hessian of the function u, and $K(x)$ denotes the Gauss curvature of the graph of u at $(x,u(x))$, which we shall assume is positive in Ω.

We start with a necessary condition for the existence of a convex $C^2(\Omega)$ solution of (1). If u is such a solution, then the gradient mapping $Du : \Omega \to \mathbb{R}^n$ is one to one with Jacobian $\det D^2u$, so by integrating (1) we obtain

$$\begin{aligned}\int_\Omega K &= \int_\Omega \frac{\det D^2u}{(1+|Du|^2)^{(n+2)/2}} \\ &= \int_{Du(\Omega)} \frac{dp}{(1+|p|^2)^{(n+2)/2}} \\ &\le \int_{\mathbb{R}^n} \frac{dp}{(1+|p|^2)^{(n+2)/2}} \\ &= \omega_n ,\end{aligned}$$

where ω_n is the measure of the unit ball in $\mathbb{R}^n$. Thus the condition

(2) $$\int_\Omega K \le \omega_n$$

is necessary for the existence of a convex solution $u \in C^2(\Omega)$ of (1).