NON-LINEAR CHARACTERIZATIONS OF SUPERREFLEXIVE SPACES

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1. The classical theorem of Weierstrass on approximation says that a real continuous function on a closed, bounded set in a finite dimensional space is the limit of a uniformly convergent sequence of polynomials. While this theorem has very interesting extensions, such as the Stone-Weierstrass Theorem, it does not generalise in this form to infinite dimensional spaces. A.S. Nemirovski and S.M. Semenov [5] have given an example of a real continuous function on a separable, infinite Hilbert space H, possessing uniformly continuous Fréchet derivatives of all orders but, which, on the unit ball of H cannot be approximated uniformly by polynomials. However, they show that every uniformly continuous function on the unit ball of H is the uniform limit of restrictions of functions which are uniformly continuously differentiable on bounded sets. For a discussion of these results see [7]. Results of this type in global analysis on infinite dimensional manifolds raise the question of existence of uniformly continuously differentiable functions on a Banach space which have bounded support. R. Bonic and J. Frampton [2] studied questions of similar nature. If X and Y are Banach spaces, let $C^{p,q}(X,Y)$, $0 \leq q \leq p \leq \infty$, denote those functions in $C^p(X,Y)$ whose derivatives of order less than or equal to q are bounded. Call a Banach space X, $C^{p,q}$-smooth if there exists a nonzero $C^{p,q}$-function on X with bounded support. In this notation, finite dimensional spaces are $C^{\infty,\infty}$-smooth and if an L_p space is C^p-smooth, then it is also $C^{p,q}$-smooth. Consider the space c_0 of all real bounded