A REMARK ON FULLY NONLINEAR, CONCAVE ELLIPTIC EQUATIONS

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0. INTRODUCTION AND STATEMENT OF THE RESULT

In this note we shall be concerned with fully nonlinear elliptic equations of second order of the form

$$(1) \qquad F(D^2u) = g(x)$$

for solutions $u(x) \in C^4(\Omega)$, defined in an open subset Ω of $\mathbb{R}^n$ $(n \geqslant 2)$. Here $F \in C^2(\mathbb{R}^{n\times n})$ and $g \in C^2(\Omega)$, with $\mathbb{R}^{n\times n}$ denoting the space of symmetric $n\times n$ matrices $r = [r_{ij}]$. We shall impose the following *assumptions*:

(i) F is uniformly elliptic for u , that is, there exist positive constants λ,Λ such that

$$\lambda|\xi|^2 \leqslant F_{r_{ij}}(D^2u)\xi_i\xi_j \leqslant \Lambda|\xi|^2$$

for all $\xi \in \mathbb{R}^n$.

(ii) F is a concave function on some convex set containing the range of D^2u , so that

$$F_{r_{ij}r_{k\ell}}\eta_{ij}\eta_{k\ell} \leqslant 0$$

for all $\eta = [\eta_{ij}] \in \mathbb{R}^{n\times n}$.

(iii) In addition

$$|g|_{2;\Omega} \leqslant K\,, \qquad |u|_{2;\Omega} \leqslant M$$

for some constants K,M .