

ULTRAPOWERS IN THE LIPSCHITZ AND UNIFORM
CLASSIFICATION OF BANACH SPACES

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Ultraproducts were introduced into Model Theory by Skolem in the 1930s. For some time they have been used in algebra. Their use in Analysis commenced in the 1970s. A major contribution was made by Stern in 1978 when he derived the Banach space versions of the Lowenheim-Skolem Theorem and the Keisler-Shelah Theorems which appear in this paper. More recently Heinrich and Mankiewicz have made a significant contribution to the use of ultrapowers in the Lipschitz and Uniform Classification of Banach spaces. They considered various Banach spaces (with certain natural properties) which were related by some uniform or Lipschitz mapping. Using Ultrapower techniques many useful results were obtained, including greatly simplified proofs of some difficult results of Ribe [1976], [1978].

In many cases the existence of some (non-linear) uniform or Lipschitz mapping between two Banach spaces (with certain additional properties) guarantees the existence of a linear mapping between them.

In this paper the intention is to provide a look at some of the most critical results in this area, to provide a feeling for the use of them, and to finally prove the result that the ultrapowers of certain uniformly equivalent Banach spaces are in fact linearly equivalent.

FILTERS

A *filter* on an index set I is a non-empty family \mathcal{F} of subsets of I which is