ON AN ELLIPTIC BOUNDARY VALUE PROBLEM WITH MIXED NON-LINEAR BOUNDARY CONDITIONS

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1. INTRODUCTION

E. Tuck has made a study of airflows (assumed to be irrotational of an inviscid incompressible fluid) under a thin body at a non-uniform small clearance from a plane ground surface. (See [6]). The problem is relevant to vehicle aerodynamics, especially for racing cars.

J. van der Hoek and the present author have begun an investigation whose immediate aim is to establish existence, uniqueness and regularity properties for the model used by Tuck. This paper is a report of some of that work.

We take the body to be fixed and the flow to have a uniform velocity at infinity of 1 in the positive x-direction. The plan form of the body is assumed to be a bounded convex domain Ω in $\mathbb{R}^2$ which is symmetric with respect to the x-axis and has a smooth boundary $\partial\Omega$. The height of the body above the ground surface is given by $z = a(x,y)$ where a is a positive smooth function on $\overline{\Omega}$ satisfying $a(x,y) = a(x,-y)$. Let $\varphi(x,y,z)$ be the velocity potential of the flow so that $\varphi(x,y,z) = x$ at infinity. For points $q \in \partial\Omega$ let $\nu(q)$ be the outward pointing unit normal to $\partial\Omega$ and $\tau(q)$ the clockwise pointing unit tangent. The boundary decomposes in the form $\partial\Omega = \Gamma_L \cup \Gamma_T \cup \{p,\overline{p}\}$ where Γ_L (the leading edge) and Γ_T (the trailing