PHASE RETRIEVAL AS A NONLINEAR ILL-POSED PROBLEM

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INTRODUCTION

Phase retrieval is a common problem in many branches of physics, such as optics or crystallography, but a great deal of difficulty has been encountered in the construction of numerical solutions to it. This arises because the problem is ill-posed; this talk briefly describes the two sources of ill-conditioning, nonuniqueness and discontinuous dependence of the solution on the data, and considers the implication for numerical algorithms for the problem's solution.

The prototypical phase retrieval problem occurs when a light beam passes through a small aperture B and then falls on a flat screen A. Classical optics states that the wavefront at A is the Fourier transform of the wavefront across the aperture, so that knowledge of the wavefront in the plane of A allows reconstruction of it at B. Measurement of the intensity of the beam on the screen is an easy task; measurement of the phase is quite a different matter and is usually impossible. This leads to the following mathematical model problem of phase retrieval

> Given the measured modulus $m(s)$ of a function $g(s)$ on a bounded set $A \subset \mathbb{R}^N$, where $g(s)$ is the Fourier transform of a function $G(w)$ with support contained in the bounded set $B \subset \mathbb{R}^N$, find