ASYMPTOTICALLY STABLE SOLUTIONS OF THE NAVIER-STOKES EQUATIONS AND ITS GALERKIN APPROXIMATIONS

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In many numerical and theoretical studies in fluid dynamics, especially in meteorology and oceanography, simpler truncated systems called Galerkin approximations or spectral systems, are studied instead of the full system of partial differential equations. These are finite dimensional systems of ordinary differential equations, usually with only linear and quadratic terms, which are obtained by truncating infinite dimensional systems involving the time-dependent coefficients of Fourier-like series expansions of the solutions of the partial differential equations. An implicit assumption here is that the qualitative behaviour of the solutions of the truncated system closely resemble that of the solutions of the full system of partial differential equations. This is known not to be true, in the Lorenz equations for example, when the truncation is too severe or the type of behaviour under consideration too complicated. It is, however known from the work of Foias, Prodi and Temam [2,3,4] that a compact attracting set for the Navier-Stokes equations is essentially finite-dimensional. In addition Constantin, Foias and Temam [1] have recently shown for the Navier-Stokes equations that the presence of an asymptotically stable steady solution in a Galerkin approximation, defined in terms of the eigenfunctions of the Stokes operator, of sufficiently high order implies the existence of a nearby asymptotically stable solution in the full Navier-Stokes equations. Their proof makes considerable use of the spectral properties of the linear operators in the Galerkin approximations and the Navier-Stokes equations linearized about steady solutions.