POWER CONCAVITY OF SOLUTIONS OF DIRICHLET PROBLEMS

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This talk consists of two items. The first is a simplified version of a concavity theorem. The second is an indication of how the result might be extended to a certain class of Dirichlet problems for degenerate quasilinear equations.

Let Ω be a bounded convex domain in $\mathbb{R}^n$ with $n \geqslant 2$. It was recently shown ([2]), under some complicated conditions on the positive function $b\colon \Omega \times (0,\infty) \times \mathbb{R}^n \to \mathbb{R}$, that if $u \in C(\overline{\Omega}) \cap C^2(\Omega)$ is a solution to the problem

$$\Delta u + b(x,u,Du) = 0 \quad \text{in } \Omega$$
$$u = 0 \quad \text{on } \partial\Omega,$$

then u is power concave. That is, $u(x)^\alpha$ is a concave function of x in $\overline{\Omega}$ for some $\alpha > 0$. The conditions stated for b were inequalities for various polynomials of derivatives of b. These conditions were difficult to interpret, but have now been considerably simplified, as shown in case (7) of the following table, which summarises power concavity results for various categories of function b. For any category C of function b, let

$$\alpha_0(C) = \inf \{\alpha \in \mathbb{R};\ (u(x)^\alpha - 1)/\alpha \text{ is concave in } \Omega\}$$

where the infimum is taken over all b in C and all bounded convex Ω, and $(u(x)^\alpha-1)/\alpha$ is understood to mean $\log(u(x))$ when $\alpha = 0$. [Note: the above set of α is an interval, since if $(u^\alpha-1)/\alpha$ is concave, then $(u^\beta-1)/\beta$ is concave for all $\beta \leqslant \alpha$.]