MINIMISING CURVATURE — A HIGHER DIMENSIONAL ANALOGUE OF THE PLATEAU PROBLEM

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The classical problem at the heart of contemporary geometric measure theory is the Plateau problem. One is given a smooth compact $(k-1)$-dimensional manifold ("boundary") B in $\mathbb{R}^n$ and one asks whether there is a k dimensional object M with boundary ∂M equal to B and having least k-dimensional volume among all such objects.

In order to make the above problem precise we need to clarify in particular the following notions: k dimensional object; k dimensional volume; boundary. In order to solve the problem by means of the usual variational approach (i.e. take a minimising sequence, extract a convergent subsequence, and show the limit has the required properties) our class of k dimensional objects must carry a topology which gives the required compactness and lower semi continuity properties. In a landmark paper [FF], Federer and Fleming introduced the class of k-dimensional integer multiplicity currents in $\mathbb{R}^n$, proved the appropriate compactness property (very difficult) and semi continuity property, and solved the Plateau problem in this context (see the references [F] and [S] for details and further references).

In order to fix our ideas we remark that we can represent a k dimensional integer multiplicity current T in $\mathbb{R}^n$ as a triple $\underline{t}(M,\theta,\xi)$ where M is a countably k-rectifiable subset of $\mathbb{R}^n$, θ is a $\mathcal{H}^k$ measurable summable non-negative integer valued function defined over M, and ξ is a $\mathcal{H}^k$ measurable function defined over M which assigns to $\mathcal{H}^k$ a.e. $x \in M$ one of the two possible orientations of the approximate tangent