FLOW BY MEAN CURVATURE OF CONVEX SURFACES INTO SPHERES

Gerhard Huisken

In this talk we consider a uniformly convex n-dimensional $(n \geq 2)$ surface M, which is smoothly imbedded in $\mathbb{R}^{n+1}$. Let us assume that M is locally given by a diffeomorphism

$$F_0 : U \subset \mathbb{R}^n \longrightarrow F_0(U) \subset M \subset \mathbb{R}^{n+1} .$$

Then we want to find a whole family of diffeomorphisms $F(\cdot,t)$ satisfying the evolution equation

$$\begin{aligned} &\frac{\partial}{\partial t} F(\vec{x},t) = \Delta_t F(\vec{x},t) , \quad \vec{x} \in U \\ &F(\cdot,0) = F_0 \end{aligned} \qquad (1)$$

where Δ_t is the Laplace-Beltrami operator on the manifold M_t, which is given by $F(\cdot,t)$. We have

$$\Delta_t F(\vec{x},t) = -H(\vec{x},t)\cdot\nu(\vec{x},t)$$

where $H(\cdot,t)$ is the (positive) mean curvature and $\nu(\vec{x},t)$ the (outer) unit normal on M_t : The surfaces M_t are moving along their mean curvature vector. Since problem (1) is parabolic, we know that it has a smooth solution at least on some short time interval.