Smooth Foliations Generated by Functions of Least Gradient

by

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The work that is outlined below has been done jointly with Harold Parks, Oregon State University.

Let $\Omega \subset R^n$ be a bounded open set and suppose $u \in BV(\Omega)$. The function is said to be of least gradient with respect to Ω if for each $v \in BV(\Omega)$ such that $u = v$ outside some compact subset of Ω,

$$\int_\Omega |\nabla u| \leq \int_\Omega |\nabla v| .$$

A function of least gradient need not be continuous. Indeed, for any subset $A \subset \Omega$, the portion of the reduced boundary of A which lies in Ω is area minimizing if and only if the characteristic function of A is of least gradient.

In this work we consider the question of regularity of functions of least gradient subject to boundary constraints. Thus, we consider an open, bounded set $\Omega \subset R^n$ that is uniformly convex. We also assume that Ω is smoothly (C^∞) bounded. Let $\phi : \text{bdry}\ \Omega \to R^1$ be smooth and consider the variational problem

(1) $$\inf \left\{ \int_\Omega |\nabla u| : u = \phi \text{ on bdry } \Omega \right\}$$

where the infimum is taken over all Lipschitzian u. It was shown in [PH1], [PH2] that the variational problem (1) admits a unique extremal. The Euler-