BOUNDARY VALUE PROBLEMS FOR FULLY NONLINEAR ELLIPTIC EQUATIONS

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We describe here some recent estimates and existence theorems for classical solutions of nonlinear, second order elliptic boundary value problems of the general form,

(1) $$F[u] = F(x,u,Du,D^2u) = 0 \quad \text{in} \quad \Omega ,$$

(2) $$G[u] = G(x,u,Du) = 0 \quad \text{on} \quad \partial\Omega ,$$

where Ω is a bounded domain in Euclidean n space, $\mathbb{R}^n$, F, G are real valued functions on the sets $\Gamma = \Omega \times \mathbb{R} \times \mathbb{R}^n \times \mathbb{S}^n$, $\Gamma' = \partial\Omega \times \mathbb{R} \times \mathbb{R}^n$ respectively and $\mathbb{S}^n$ denotes the linear space of $n \times n$ symmetric real matrices. Letting $X = (x,z,p,r)$, $X' = (x,z,p)$ denote points in Γ, Γ', we adopt the following definitions of ellipticity and obliqueness for functions, F,G differentiable with respect to r, p respectively. Namely, the operator F is *elliptic* at $X \in \Gamma$ if the matrix

$$F_r = [F^{ij}] = [\frac{\partial F}{\partial r_{ij}}]$$

is positive at X; while the operator G is *oblique* at $X' \in \Gamma'$ if

$$\chi = G_p = \nu \cdot G_p$$