HAMILTONIAN SYSTEMS WITH MONOTONE TRAJECTORIES

John Toland

1. INTRODUCTION

Recently Helmut Hofer and I [1] studied a class of ordinary differential equations on $\mathbb{R}^{2n}$ which possess a Hamiltonian structure of a non-standard type. We considered the class of equations which can be written in the form

$$\dot{q}(t) = Sp(t) \ , \quad \dot{p}(t) = -V'(q(t)) \ , \quad (q(t),p(t)) \in \mathbb{R}^n \times \mathbb{R}^n \ , \ t \in \mathbb{R} \ ,$$

where S is a non-singular invertible self-adjoint operator on $\mathbb{R}^n$ with one negative and $n-1$ positive eigenvalues, and V' denotes the gradient of a smooth potential function V. The total energy, or Hamiltonian, which is conserved along trajectories is

$$H(q(t),p(t)) = \frac{1}{2}\,(Sp(t),p(t)) + V(q(t)) \ .$$

Because S is not positive-definite the quadratic form $(Sp(t),p(t))$ may be negative, and since the corresponding quadratic form in classical Hamiltonian dynamics is the positive kinetic energy functional, our theory does not include classical particle dynamics as a special case. However, systems such as ours do arise in applied mathematics, the most familiar example being the nonlinear Sturm Liouville problems for equations and systems.