ISOLATED SINGULARITIES FOR EXTREMA OF GEOMETRIC VARIATIONAL PROBLEMS

Leon Simon

We here want to consider asymptotic behaviour on approach to an isolated singularity of an extremal u of a functional $\mathcal{F}(u)$ of the form

$$(*) \qquad \mathcal{F}(u) = \int_{B_1(0)} F(x,u,Du)\,dx ,$$

where F is a given function and $B_1(0)$ is the open unit ball in $\mathbb{R}^n$. u is allowed to be vector-valued with values $u(x) = (u^1(x),\ldots,u^N(x)) \in \mathbb{R}^N$. What we have to say here has a natural generalization to the case when the domain of integration $B_1(0)$ in $(*)$ is replaced by a conical domain C_1 of the form $\{\lambda w: 0<\lambda<1,\ \omega\in\Sigma\}$, where Σ is some smooth embedded submanifold of S^{n-1}, and also to the case when $u(x) = u(r\omega)$ $(r=|x|, \omega=x/|x|)$ is a section of some vector bundle over Σ for each fixed r. For these generalizations (which are important, for example, for applications to minimal submanifolds) we refer to the paper [SL1]. In any case the essential ideas are the same in this less general setting.

Our main aim is to discuss asymptotic behaviour of an extremal $u = u(r\omega)$ of $(*)$ as $r\downarrow 0$, in case u has an isolated discontinuity at 0; notice that by an extremal of $\mathcal{F}(u)$ we mean a function u which satisfies the Euler-Lagrange system of $(*)$ in $B_1(0) \sim \{0\}$; thus u satisfies

$$(1) \qquad \mathcal{N}u = 0 \quad \text{in} \quad B_1(0) \sim \{0\} ,$$

where $\mathcal{N}u$ is the second order quasilinear operator (with values in $\mathbb{R}^N$)