# ON SUFFICIENT CONDITIONS FOR OPTIMALITY

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Let $f, g_1, \ldots, g_m$ be continuously differentiable real valued functions defined on a domain $\Omega$ of n-dimensional real space $\mathbb{R}^n$. We consider the following optimization problem.

$$f(x) \to \inf$$

(1)

$$g_i(x) \le 0 \, , \quad x \in \Omega \, .$$

Let $x_0 \in \Omega$. We assume that at $x_0$ all constraints $g_i$ are active, i.e. $g_i(x_0) = 0$.

THEOREM 1 ([9]): *Suppose that at the point $x_0$ all gradients of $g_i$, $\nabla g_i$, are linearly independent. Suppose that at $x_0$ Kuhn-Tucker necessary conditions for optimality hold, i.e. there are $\lambda_i \ge 0$ such that*

(2)
$$\nabla (f + \Sigma \lambda_i g_i) \Big|_{x_0} = 0 \, .$$

*If all $\lambda_i > 0$, $i = 1,2,\ldots,m$, then $x_0$ is a local minimum of problem* (1) *if and only if it is a local minimum of the following equality problem*

$$f(x) \to \inf$$

(3)

$$g_i(x) = 0 \, .$$

The proof of Theorem 1 is elementary and uses only the implicit functions theorem. Theorem 1 gives a very useful algorithm for reducing a problem of sufficient condition for problem (1) to well-known classical

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