MINIMUM PROBLEMS FOR NONCONVEX INTEGRALS

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1. INTRODUCTION

Let us consider an integral of the Calculus of Variations of the following type :

$$(1.1) \qquad F(u;\Omega) = \int_\Omega f(x,u(x),Du(x))dx ,$$

where Ω is a bounded open set in $\mathbb{R}^n$, $u : \Omega \to \mathbb{R}^m$ is a function belonging to $W^{1,p}(\Omega;\mathbb{R}^m)$, $p>1$ and $f(x,u,\xi)$ is a Carathéodory function, i.e. measurable with respect to x, continuous in (u,ξ). The direct method to get the existence of minima for the Dirichlet problem

$$(P) \qquad \text{Inf } \{F(u;\Omega) \; : \; u-u_0 \in W_0^{1,p}(\Omega; \mathbb{R}^m)\} ,$$

where u_0 is a fixed function in $W^{1,p}$, is based on the sequential lower semicontinuity of F(s.l.s.c.) in the weak topology of $W^{1,p}$.

If $m=1$, it is well known (see [7],[8],[10]) that the l.s.c. of F is equivalent, under very general growth assumptions on f, to the condition that the integrand is a convex function of the variable ξ. But if $m>1$, convexity is no longer a necessary condition. To see this, let us consider a continuous function $f : \mathbb{R}^{mn} \to \mathbb{R}$ such that the functional $\int_\Omega f(Du(x))dx$ is weakly*