APPROXIMATION BY COMPACT OPERATORS BETWEEN CLASSICAL FUNCTION SPACES

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Interest in approximating a bounded linear operator T on a Hilbert space H originated with Gohberg and Krein [7, Section II.7]. They showed, constructively, that there is always a compact operator C which minimizes $\|T - C\|$. In contemporary terminology, the compact operators $K(H)$ form a proximinal subspace of $B(H)$. Another constructive proof of this fact was later given by Holmes and Kripke [9], and a comparison of the two constructions was made by Bouldin [4]. An abstract proof has also been given by Alfsen and Effros [1, Corollary 5.6].

More recently, various authors [2,3,11,12, 13, 16] have considered this problem for operators between general Banach spaces E and F. For which E and F is $K(E,F)$ a proximinal subspace of $B(E,F)$? In this expository talk, we will summarize what is known when E and F are classical function spaces - that is, $C(X)$, where X is compact and Hausdorff, $L_p(\mu)$ where $1 \leq p < \infty$, or the sequence space c_0. There is no need to consider $L_\infty(\mu)$ since every such space is isometric to some $C(X)$. It will, of course, be necessary to distinguish the cases $p = 1$ and $p > 1$. Our first result establishes proximinality in the case $F = c_0$. We remark that this is nontrivial, since $K(E,c_0)$ is always a proper subspace of $B(E,c_0)$, when E is infinite dimensional, by [10] or [14].