UNCERTAINTY PRINCIPLES AND INTERFERENCE PATTERNS

John F. Price

The classical Heisenberg uncertainty principle

$$\Delta q \Delta p \geq \hbar/2 \tag{1}$$

has been one of the key relationships in quantum mechanics for over fifty years. It does have a number of weaknesses, however, particularly related to the fact that the standard deviations Δq and Δp only give very general information about the spreads of the probability density functions of position and momentum respectively. This paper surveys a number of recent inequalities which describe more subtle relationships between position and momentum or, in mathematical terms, between a function and its Fourier transform. For example, local uncertainty principle inequalities assert that if the uncertainty of momentum Δp is small, then not only is the uncertainty of position Δq large, but the probability of the system being localized at any point is also small.

So as to add a little more interest, I have applied in turn each of the inequalities, starting with (1), to the proposition by Niels Bohr that in the double-slit experiment you can have an interference pattern or know the paths of the particles, but not both. In some ways I could not have chosen a worse test-case since it turns out that for this example they are all out-performed by Poisson summation. Nevertheless it does provide an opportunity to display and contrast some of their features. Also in the end we arrive at a rigorous justification of Bohr's original argument which apparently is new.