ERROR ESTIMATES FOR A FIRST KIND INTEGRAL EQUATION AND AN ASSOCIATED BOUNDARY VALUE PROBLEM

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1. INTRODUCTION

This paper deals with an integral equation method for obtaining numerical solutions to the two-dimensional interior and exterior Dirichlet problem

(1.1a) $$\Delta U = 0 \qquad \text{on } \mathbb{R}^2\backslash\Gamma$$

(1.1b) $$U = g \qquad \text{on } \Gamma$$

(1.1c) $$U(X) = O(1) \qquad \text{as } |X| \to \infty .$$

Here

$$\Delta = \left(\frac{\partial}{\partial X_1}\right)^2 + \left(\frac{\partial}{\partial X_2}\right)^2 , \qquad X = (X_1, X_2) \in \mathbb{R}^2$$

is the Laplacian, Γ is a simple closed C^∞ curve in the plane, and g is a given function on Γ. It will sometimes be necessary to refer to the bounded and unbounded components of $\mathbb{R}^2\backslash\Gamma$, and these will be denoted by Ω_+ and Ω_- respectively. Also, $\nu(Y)$ denotes the unit normal at $Y \in \Gamma$ pointing into Ω_+, and σ denotes the arc length measure on Γ.

The boundary value problem (1.1) can be reduced to an integral equation on Γ by seeking a representation of the solution in the form of a single layer potential

(1.2) $$U(X) = \frac{1}{\pi}\int_\Gamma \log\left(\frac{1}{|X-Y|}\right) v(Y)\, d\sigma(Y) + \omega , \qquad X \in \mathbb{R}^2 ,$$

with v an unknown function on Γ and ω an unknown constant. If $v \in L_p(\Gamma)$ for some $p > 1$ then the formula (1.2) defines a function