THE SOLUTION OF SYSTEMS OF OPERATOR EQUATIONS USING CLIFFORD ALGEBRAS

Alan McIntosh and Alan Pryde

1. INTRODUCTION

Our aim is twofold. We develop a functional calculus for commuting m-tuples of Banach space operators, and then use this functional calculus to solve a system of operator equations and obtain estimates for the solution. The new ingredient is the use of Clifford algebras.

As a corollary we obtain results on the perturbation of the spectral subspaces of commuting self-adjoint operators. In particular we answer an open question, stated for example on p. 221 of [5], on the spectral perturbation of self-adjoint matrices.

Our idea of using Clifford algebras is derived from the work of R. Coifman and M. Murray [10]. The functional calculus for several operators is a generalization of that developed in S. Kantorovitz [7] and I. Colojoara and C. Foiaş [4] for a single operator. Our results on systems of operator equations extend results of R. Bhatia, Ch. Davis and A. McIntosh [2] concerning single equations. Thanks are due to J. Picton-Warlow with whom we have had several stimulating discussions.

Banach spaces X and Hilbert spaces H and K are defined over the field $\mathbb{F}$, where $\mathbb{F}$ denotes either the real field $\mathbb{R}$ or the complex field $\mathbb{C}$.

2. OPERATOR EQUATIONS

To motivate our discussion of the functional calculus, we state here our results on systems of operator equations.

Throughout this section, $\underset{\sim}{A} = (A_1,\dots,A_m)$ and $\underset{\sim}{B} = (B_1,\dots,B_m)$ denote commuting m-tuples of bounded self-adjoint operators defined on Hilbert spaces H and K respectively. The joint spectrum of $\underset{\sim}{A}$ is denoted $\sigma(\underset{\sim}{A})$.