

SOME THEOREMS ON ORLICZ-SOBOLEV SPACES,  
AND AN APPLICATION TO NEMITSKY OPERATORS

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1. INTRODUCTION

We are concerned here with the problem of extending, to Orlicz-Sobolev spaces, certain theorems of Marcus and Mizel on Nemitsky operators on Sobolev spaces. (See [5].)

Marcus and Mizel's proofs rely upon, in particular,

- (i) Gagliardo's characterisation of the Sobolev space  $W_{1,p}$  in terms of absolute continuity; and
- (ii) bounds and limits of difference quotients in Sobolev spaces.

We shall give suitable extensions of (i) and (ii) to Orlicz-Sobolev spaces in §§ (2) and (3) below, which enables us to give an extension of the theorems of Marcus and Mizel. (See § 4.)

2. ORLICZ-SOBOLEV SPACES AND THE SPACES  $A(\Omega)$

Throughout this paper,  $\Omega$  denotes a domain in  $R^n$ .

Since all the definitions of both Orlicz and Orlicz-Sobolev spaces which occur in the statements of our theorems can be found in [1], we shall not repeat them here. For the spaces  $A(\Omega)$  (i.e., Beppo Levi spaces), we shall follow [5]. (With a minor difference in notation, essentially that, in denoting certain equivalence classes, we use "~" instead of a dash, to avoid an obvious source of confusion.)