

TRANSFERRING FOURIER MULTIPLIERS

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1. FOURIER MULTIPLIERS OF $L^p(G)$

Let G be a compact Lie group, and \hat{G} is dual (a maximal set of irreducible representations of G). The Fourier transform of

$f \in L^1(G)$ associates to $\sigma \in \hat{G}$, the $d_\sigma \times d_\sigma$ matrix

$\int_G f(x) \sigma(x^{-1}) dx$ (where d_σ is the dimension of the space in which σ acts).

The Fourier multipliers of $L^p(G)$ are sequences (A_σ) of matrices so that if $(\hat{f}(\sigma))$ is the Fourier series of an L^p function, so is $(A_\sigma \hat{f}(\sigma))$.

Example. If $G = SU(2)$, $\hat{G} \equiv \{0, \frac{1}{2}, 1, \dots\}$ and if $\ell \in \hat{G}$, σ_ℓ has dimension $2\ell+1$, and we look for sequences $A_0, A_{\frac{1}{2}}, \dots$, where A_ℓ is a $(2\ell+1) \times (2\ell+1)$ matrix.

2. EXAMPLES OF MULTIPLIERS

(i) Central multipliers. We restrict to $A_\sigma = c_\sigma I$ for $c_\sigma \in \mathbb{C}$. This is the case which has been most studied. For example, Bonami and Clere [1] and Clere [2] have shown that the

$$\begin{aligned} \text{Poisson kernel} & e^{-\sqrt{\frac{\ell}{R}}} I_{\sigma_\ell} \\ \text{Gauss kernel} & e^{-\frac{\ell}{R}} I_{\sigma_\ell} \\ \text{Riesz kernel} & \left(1 - \frac{\ell}{R}\right)^\delta + I_{\sigma_\ell} \quad (\delta > 1) \end{aligned}$$

are bounded summability kernels in $L^p(SU(2))$. These results also