GROUP ACTIONS ON CUNTZ ALGEBRAS

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1. INTRODUCTION

The Cuntz algebra $\mathcal{O}_n$ $(1 < n < \infty)$ is the C* -algebra generated by the range of a linear map s from C^n to the bounded linear operators on an infinite dimensional Hilbert space which satisfies

(1.1) $$s(h_1)^*s(h_2) = \langle h_1, h_2 \rangle 1 , \quad h_j \in C^n , j = 1,2$$

(1.2) $$\Sigma_{j=1,n} s(e_j)s(e_j)^* = 1 ,$$

where $\langle , \rangle$ is an inner product on C^n, $\{e_j\}_{j=1,n}$ an orthornormal basis with respect to this inner product and 1 the identity operator. One may think of $\mathcal{O}_n$ as a 'non-commutative version' of the unit sphere in C^n . This analogy is reinforced by the fact that the noncompact lie group $U(n,1)$ acts automorphically on $\mathcal{O}_n$ by generalised Mobius transformations. This $U(n,1)$ action was introduced by Voiculescu [6], however, understanding his proof of its existence requires some stamina on the part of the reader. We show here that the action may be defined using just elementary algebra and the result of Cuntz [3] that $\mathcal{O}_n$ is uniquely determined by the relations (1.1) and (1.2) satisfied by s .

2. THE U(n,1) ACTION

Define a row vector $s = (s(e_1),\ldots,s(e_n))$. Then with s^* denoting the column vector with entries $s(e_j)^*$ $(j = 1,\ldots,n)$ one has from (1.1) and (1.2) the relations

(2.1) $$ss^* = 1 , \quad s^*s = \mathrm{diag}(1,\ldots,1)$$

If A, B are $n \times n$ matrices over C and sAs^* denotes the obvious matrix product then