AN APPROXIMATION THEOREM FOR ORDER BOUNDED OPERATORS

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The object of this paper is to outline some recent work with P.G. Dodds, B. de Pagter and A.R. Schep [1]. In the following E and F will be Riesz spaces and T will be a positive operator from E to F. For proofs which are not given the reader is referred to a forthcoming paper [1]. Our aim is to approximate in a purely order theoretic way any operator in the order interval $[0,T]$ of the space of all regular operators between E and F with operators of a particularly simple kind with respect to T. For the sake of convenience we will assume that $E = C(K)$ (except in corollary 7), that the normal integrals on F, denoted $F_n^{\sim}$, separate the points of F and that F is Dedekind complete. The latter has as a consequence that the space of all order bounded (= regular) operators from E to F, denoted by $L_b(E,F)$ is itself a Dedekind complete Riesz space.

Every element $f \in C(K)$ determines a multiplication operator $g \rightarrow gf$ on $C(K)$, which is called a multiplier. Abstractly such operators $\sigma : C(K) \rightarrow C(K)$ are defined by the conditions that $|\sigma(g)| \wedge |h| = 0$ whenever $|g| \wedge |h| = 0$ and that σ is order bounded.

We are interested in the set of all operators R in $[0,T]$ for which there exist $n \in \mathbb{N}$, multipliers $\sigma_1, \ldots \sigma_n$ on $C(K)$ and order projections $\pi_1, \ldots \pi_n$ on F such that $R = \sum_{i=1}^{n} \pi_i T \sigma_i$. The set of all those operators will be labelled $\ell(T)$. The elements of $\ell(T)$ serve as approximating operators in $[0,T]$.

The following terminology is needed. If L is a Riesz space and